PowerLab
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TECHNICAL WORKING PAPER

Non-Linear Coefficient of Performance (COP) Degradation, Defrost Entropy Losses, and Auxiliary Resistive Staging Dynamics in Cold-Climate Air-Source Heat Pumps (ccASHP)

A Thermodynamic Framework, Building Envelope Equilibrium Model, and Electrical Demand Analysis under AHRI 210/240 and ASHRAE Fundamentals

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ABSTRACT

The rapid global transition toward residential building decarbonization has established Air-Source Heat Pumps (ASHPs) and Cold-Climate Air-Source Heat Pumps (ccASHPs) as primary heating equipment. While heat pumps exhibit outstanding nominal efficiency under standard test conditions ($\mathrm{COP} \geq 3.5$ at $47^{\circ}\mathrm{F}/8.3^{\circ}\mathrm{C}$), real-world cold-weather performance exhibits severe, non-linear degradation. This divergence creates widespread consumer "bill shock" and extreme winter peak grid stress.

This paper formulates a deterministic thermodynamic and building physics framework analyzing four interlinked mechanisms: (1) Carnot lift and refrigerant suction vapor density rarefaction (ρ_{suction}); (2) parasitic reverse-cycle defrost entropy penalties ($28^{\circ}\mathrm{F} \leq T_{\text{amb}} \leq 43^{\circ}\mathrm{F}$); (3) thermal balance point (T_{balance}) envelope inversion; and (4) the auxiliary resistive "strip heat cliff" causing instantaneous 400% to 750% electrical power demand surges.

Keywords: Cold-Climate Heat Pump, COP Decay, Suction Density, Defrost Penalty, Thermal Balance Point, Auxiliary Strip Heat, AHRI 210/240, ASHRAE Bin Method.

1. Introduction & The Electrification Dilemma

Air-source heat pumps (ASHPs) transfer heat from ambient outdoor air into conditioned indoor spaces via a closed vapor-compression refrigeration cycle. Modern inverter systems achieve rated COPs between 3.0 and 4.5 at rating conditions ($47^{\circ}\mathrm{F}/8.3^{\circ}\mathrm{C}$). However, a fundamental thermodynamic dichotomy governs building electrification: **building heating demand increases linearly as outdoor temperature plummets, whereas heat pump compressor capacity decreases non-linearly.**

> **The Winter Peak Electrification Dilemma:** When outdoor ambient temperatures drop during severe cold snaps, the building envelope heat loss reaches its annual maximum exactly when heat pump compressor thermal output and thermodynamic efficiency are at their lowest point.

2. Governing Thermodynamic Principles

2.1 Carnot Limit and Temperature Lift

The theoretical maximum heating efficiency for a vapor compression cycle operating between evaporator temperature T_{evap} and condenser temperature T_{cond} is governed by the Carnot heating COP:

$$\mathrm{COP}_{\text{carnot}}(T_{\text{amb}}, T_{\text{supply}}) = \frac{T_{\text{cond}}}{T_{\text{cond}} - T_{\text{evap}}} = \frac{T_{\text{supply}} + \Delta T_{\text{hex,in}}}{(T_{\text{supply}} + \Delta T_{\text{hex,in}}) - (T_{\text{amb}} - \Delta T_{\text{hex,out}})}$$

In actual equipment, second-law losses (non-isentropic compression, motor dissipation, throttling irreversibilities) yield an effective operational COP of $\mathrm{COP}_{\mathrm{actual}} = \eta_{\mathrm{carnot}} \cdot \mathrm{COP}_{\mathrm{carnot}}$, where $\eta_{\mathrm{carnot}} \approx 0.35\text{–}0.52$.

2.2 Refrigerant Suction Vapor Density Rarefaction

Compressor mass flow delivery $\dot{m}_{\mathrm{ref}}$ is constrained by suction vapor density ρ_{suction} entering the compressor cylinders:

$$\dot{m}_{\mathrm{ref}}(T_{\mathrm{amb}}) = \rho_{\mathrm{suction}} \left(P_{\mathrm{evap}}(T_{\mathrm{amb}}), T_{\mathrm{superheat}}\right) \cdot \dot{V}_{\mathrm{disp}} \cdot \eta_{\mathrm{vol}}$$

As saturated evaporating pressure collapses under sub-zero temperatures via the Clausius-Clapeyron relation, suction density falls precipitously. At $-20^\circ\mathrm{C}$ $(-4^\circ\mathrm{F})$, R-410A vapor density drops by **68.7%** relative to rating conditions, severely choking thermal output.

TABLE 1: R-410A THERMODYNAMIC STATE PROPERTIES ACROSS HEATING AMBIENT REGIMES

OUTDOOR TEMP (T_{AMB})	EVAP SAT TEMP	EVAP PRESSURE (P_{EVAP})	SUCTION DENSITY (P_{SUC})	RELATIVE MASS FLOW
+10°C (+50°F)	+3.3°C (+38°F)	872 kPa (126 psi)	35.8 kg/m³	**100.0% (Base)**
+0°C (+32°F)	-6.7°C (+20°F)	638 kPa (92 psi)	25.4 kg/m³	**70.9%**
-10°C (+14°F)	-16.7°C (+2°F)	449 kPa (65 psi)	17.3 kg/m³	**48.3%**
-20°C (-4°F)	-26.7°C (-16°F)	302 kPa (44 psi)	11.2 kg/m³	**31.3%**
-30°C (-22°F)	-36.7°C (-34°F)	194 kPa (28 psi)	6.8 kg/m³	**19.0%**

3. Reverse-Cycle Defrost Entropy Penalties

In humid atmospheric conditions between $-2°\text{C}$ and $+6°\text{C}$ ($28°\text{F}$–$43°\text{F}$), outdoor coil frost accretion forces periodic reverse-cycle defrost. The four-way valve shifts to cooling mode, extracting indoor heat to melt outdoor ice ($\Delta H_{\text{fusion}} = 334\ \text{kJ/kg}$) while staging 5–$10\ \text{kW}$ of resistive heat to temper indoor drafts.

$$\text{COP}_{\text{net}}(T_{\text{amb}}, \text{RH}) = \text{COP}_{\text{steady}}(T_{\text{amb}}) \cdot [1 - \phi_{\text{defrost}}(T_{\text{amb}}, \text{RH})]$$

During peak frosting ($32°\text{F}/85\%\ \text{RH}$), cyclic defrost reduces net delivered heating efficiency by **15% to 28%**.

4. The Thermal Balance Point & The Auxiliary "Strip Heat Cliff"

The **Thermal Balance Point (T_{balance})** is the exact outdoor temperature where building envelope heat loss equals maximum compressor heating capacity:

$$\dot{Q}_{\text{loss}}(T_{\text{balance}}) = (UA)_{\text{bldg}} \cdot (T_{\text{indoor}} - T_{\text{balance}}) = \dot{Q}_{\text{hp, max}}(T_{\text{balance}})$$

Below T_{balance}, supplemental electric resistance elements ($\text{COP}_{\text{aux}} \equiv 1.0$) stage on. Total system power demand transitions to:

$$P_{\text{total}}(T_{\text{amb}}) = \frac{\dot{Q}_{\text{hp, max}}(T_{\text{amb}})}{\text{COP}_{\text{net}}(T_{\text{amb}})} + \frac{\dot{Q}_{\text{loss}}(T_{\text{amb}}) - \dot{Q}_{\text{hp, max}}(T_{\text{amb}})}{1.0}$$

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Heating Capacity vs. Heat Loss & Auxiliary Staging
60k BTU/hr ▲                                  / [Building Heat Loss Q_loss]
           |                                 /
40k BTU/hr |   [Heat Pump Output Q_hp]      /
           |   ____________________________/ ←— Thermal Balance Point (T_bal = 24°F)
20k BTU/hr |  |                           /\____________________________
           |  |                          /  \ [Auxiliary Strip Heat: COP = 1.0]
 0 BTU/hr  |__|__________________________|__|___________________________→ Outdoor Temp
            -15°F                        24°F 47°F
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TABLE 2: ELECTRICAL DEMAND & SYSTEM NET COP FOR A 3-TON SYSTEM (36K BTU/HR NOMINAL)

OUTDOOR TEMP (T_{AMB})	BUILDING LOSS ($\dot{Q}_{\text{LOSS}}$)	COMPRESSOR OUTPUT	STRIP HEAT NEEDED	TOTAL POWER (P_{TOTAL})	NET COP
47°F (8.3°C)	18,400 BTU/hr	36,000 BTU/hr	0.0 kW	1.45 kW	**3.72**
35°F (1.7°C)	28,000 BTU/hr	31,500 BTU/hr	0.0 kW	2.82 kW	**2.91**
24°F (-4.4°C)	36,800 BTU/hr	36,800 BTU/hr	0.0 kW (Balance Point)	4.31 kW	**2.50**
10°F (-12.2°C)	48,000 BTU/hr	24,200 BTU/hr	7.0 kW (23,800 BTU/hr)	10.98 kW	**1.28**
0°F (-17.8°C)	56,000 BTU/hr	18,500 BTU/hr	11.0 kW (37,500 BTU/hr)	14.34 kW	**1.14**
-15°F (-26.1°C)	68,000 BTU/hr	12,000 BTU/hr	16.4 kW (56,000 BTU/hr)	18.70 kW	**1.06**

The 1,290% Load Spike Finding: As outdoor temperatures drop from 47°F down to -15°F, total electrical power draw surges from **1.45 kW to 18.70 kW**. This 12.9x power surge exposes distribution transformers to severe overload conditions during cold snaps.

5. ASHRAE Bin Integration & Delivered Heat Economics

Applying the ASHRAE Temperature Bin Method over annual weather distributions allows accurate computation of seasonal energy consumption and Levelized Cost of Heat (LCOH):

$$E_{\text{seasonal, elec}} = \sum_{j=1}^{N_{\text{bins}}} H_j \cdot P_{\text{total}}(T_j) \quad [\text{kWh}], \qquad \text{LCOH} = \frac{C_{\text{fuel/elec}}}{\eta_{\text{seasonal}} \cdot \text{Energy Density}} \quad \left[\frac{\$}{\text{MMBtu}}\right]$$

TABLE 3: LEVELIZED COST OF HEAT DELIVERED ($/MMBTU) ACROSS FUEL TYPES

HEATING TECHNOLOGY	UNIT FUEL PRICE	RATED EFFICIENCY	EFFECTIVE SEASONAL COP	DELIVERED COST ($/MMBTU)
ccASHP (Cold Climate Inverter)	$0.16 / kWh	Variable EVI Inverter	2.85 Seasonal	**$16.44 / MMBtu**
Standard Heat Pump (Mid-Tier)	$0.16 / kWh	Standard Staged	2.20 Seasonal	**$21.30 / MMBtu**
Natural Gas (96% Condensing)	$1.20 / Therm	96% AFUE	0.96	**$12.50 / MMBtu**
Natural Gas (80% Standard)	$1.20 / Therm	80% AFUE	0.80	**$15.00 / MMBtu**
Propane (LP Furnace)	$2.85 / Gallon	92% AFUE	0.92	**$33.91 / MMBtu**
Heating Oil (#2 Fuel Oil)	$3.90 / Gallon	85% AFUE	0.85	**$33.15 / MMBtu**
Electric Resistance Heat Strip	$0.16 / kWh	100% Joule Heating	1.00	**$46.88 / MMBtu**

6. Open Computational Implementation

The algorithms and thermodynamic models derived in this paper are openly accessible as interactive deterministic calculation engines:

- **Heat Pump Running Cost Calculator:** https://www.powelab.org/home-energy/heat-pump-cost-calculator
- **Central AC & Heat Pump Electricity Cost Calculator:** https://www.powelab.org/home-energy/air-conditioner-cost-calculator
- **Space Heater Operating Cost Calculator:** https://www.powelab.org/home-energy/space-heater-cost-calculator

7. References & Standards

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